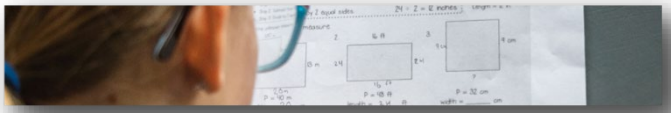
INFORMATION FOR EDUCATORS AND SCHOOL LEADERS

K-4 Foundational Mathematics Learning

BRITISH COLUMBIA | Ministry of Education and Child Care

PURPOSE

- The K-4 Math Foundational Learning Progressions (the Learning Progressions) help teachers navigate the curriculum and give guidance about what proficient students look like against the K-4 Math content Learning Standards.
- The Learning Progressions provide additional details and clarity for teachers about the Learning Standards in the Mathematics curriculum. While they are based on the curriculum, they do not replace it or teachers' professional expertise about using teaching strategies that best meet the needs of their students.
- The proficiency descriptors in the Learning Progressions align with the curriculum Learning Standards and learning goals for students at the end of the school year, from kindergarten to Grade 4. Teachers might use the Learning Progressions for planning, teaching and classroom assessment.



GUIDING INFORMATION

- Foundational mathematics skills, alongside the learning, practice, and demonstration of math curricular competencies and cross-curricular Numeracy experiences, support students in developing necessary Numeracy skills to become Educated Citizens.
- Most students develop along the proficiency scale continuum throughout the school year. Students may first be assessed as "emerging" or "developing" as they develop towards proficient in the foundational skills, as outlined by the high standards in the Learning Progressions.
- The K-4 Math Foundational Learning Progressions below are organized by skill. The Learning Progression Skills are Number Sense, Computational Fluency, Patterning and Algebraic Thinking, Geometry and Measurement, Data and Probability, and Financial Literacy. These skills align with and support the content learning standards of the Mathematics curriculum.

CURRICULUM-ALIGNED RESOURCES

- Teachers can use instructional strategies and learning resources recommended by professional associations, schools and districts, their own professional development, or outlined here, to help students develop toward end-of-year proficiency.
- See the Teaching and Learning Stories for examples of how current classroom teachers are supporting their students in developing cross-curricular Numeracy skills.

- **COMING SOON:**
  - Teaching and Learning stories showcasing foundational math skills – including samples of proficient student work
  - Further supporting resources in foundational mathematics and Numeracy instructional strategies in support of teachers' requests during the K-4 Foundational Learning Progressions Field Review

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WHAT DOES MATH LEARNING LOOK LIKE IN THE CLASSROOM?

- Teachers are professionals who choose the strategies, resources, and applications best suited to the needs of students in their local setting (e.g., embedding mathematics in issues, projects, and passions relevant to the local community).
- Teachers can focus on "hands-on" experiential learning by incorporating the development of foundational skills through opportunities to encounter math in a wide variety of situational contexts. We encourage teachers to utilize instructional strategies in which students:
  - Reason mathematically to solve problems
  - Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving
  - Recognize that there are multiple ways to solve a problem and choose and use effective strategies and tools to solve problems
  - Communicate and reflect on their thinking process and solution using mathematical vocabulary
  - Engage in problem-solving experiences that are connected to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures
- Observing, learning, and engaging in mathematical thinking empowers us to make sense of our world. Teachers who embed various ways of learning, practicing, and demonstrating mathematical knowledge and skills help students to develop as confident mathematicians who can enjoy and explore math and use math as a tool to unlock other areas of the curriculum.

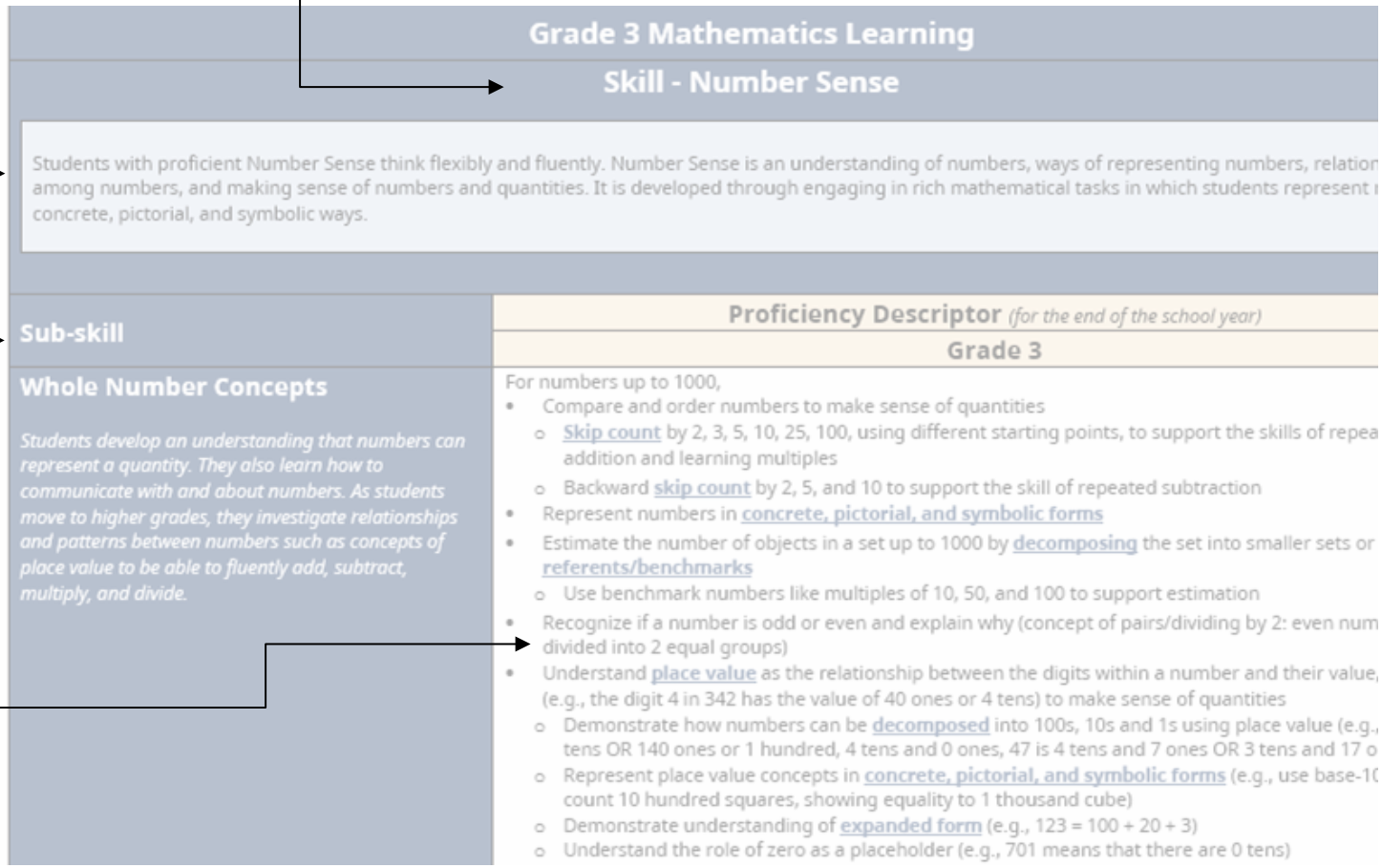
OVERVIEW

Each **SKILL** represents the ability to demonstrate foundational mathematics knowledge. A proficient student should be able to develop, practice, and demonstrate each skill. All skills are important when building knowledge in mathematics.

Each **SKILL DESCRIPTOR** provides additional information as to the development within each **SKILL**.

Each **SUB-SKILL** further defines each skill.

The **DESCRIPTOR** is grade specific and describes what proficient student learning looks like at the end of the year/term.



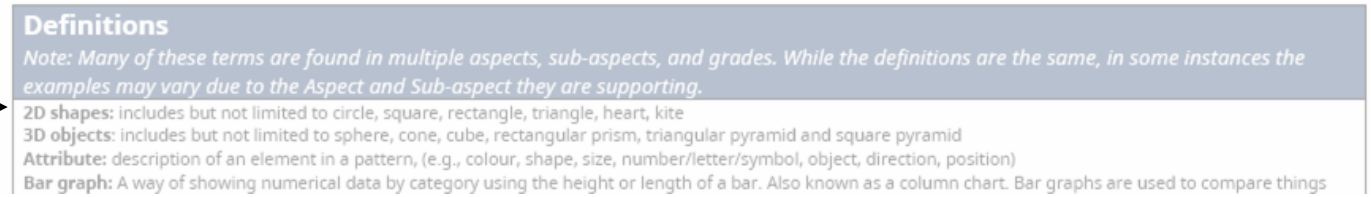
Grade 3 Mathematics Learning

Skill - Number Sense

Students with proficient Number Sense think flexibly and fluently. Number Sense is an understanding of numbers, ways of representing numbers, relation among numbers, and making sense of numbers and quantities. It is developed through engaging in rich mathematical tasks in which students represent concrete, pictorial, and symbolic ways.

Sub-skill	Proficiency Descriptor *(for the end of the school year)* Grade 3
Whole Number Concepts *Students develop an understanding that numbers can represent a quantity. They also learn how to communicate with and about numbers. As students move to higher grades, they investigate relationships and patterns between numbers such as concepts of place value to be able to fluently add, subtract, multiply, and divide.*	For numbers up to 1000, • Compare and order numbers to make sense of quantities o Skip count by 2, 3, 5, 10, 25, 100, using different starting points, to support the skills of repea addition and learning multiples o Backward skip count by 2, 5, and 10 to support the skill of repeated subtraction • Represent numbers in concrete, pictorial, and symbolic forms • Estimate the number of objects in a set up to 1000 by decomposing the set into smaller sets or referents/benchmarks o Use benchmark numbers like multiples of 10, 50, and 100 to support estimation • Recognize if a number is odd or even and explain why (concept of pairs/dividing by 2: even num divided into 2 equal groups) • Understand place value as the relationship between the digits within a number and their value, (e.g., the digit 4 in 342 has the value of 40 ones or 4 tens) to make sense of quantities o Demonstrate how numbers can be decomposed into 100s, 10s and 1s using place value (e.g., tens OR 140 ones or 1 hundred, 4 tens and 0 ones, 47 is 4 tens and 7 ones OR 3 tens and 17 o o Represent place value concepts in concrete, pictorial, and symbolic forms (e.g., use base-10 count 10 hundred squares, showing equality to 1 thousand cube) o Demonstrate understanding of expanded form (e.g., 123 = 100 + 20 + 3) o Understand the role of zero as a placeholder (e.g., 701 means that there are 0 tens)

DEFINITIONS provide further explanation for key concepts found in the **foundational proficiency descriptors**.



Definitions

Note: Many of these terms are found in multiple aspects, sub-aspects, and grades. While the definitions are the same, in some instances the examples may vary due to the Aspect and Sub-aspect they are supporting.

2D shapes: includes but not limited to circle, square, rectangle, triangle, heart, kite
3D objects: includes but not limited to sphere, cone, cube, rectangular prism, triangular pyramid and square pyramid
Attribute: description of an element in a pattern, (e.g., colour, shape, size, number/letter/symbol, object, direction, position)
Bar graph: A way of showing numerical data by category using the height or length of a bar. Also known as a column chart. Bar graphs are used to compare things

Grade 3 Mathematics Learning

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Sub-skill	Proficiency Descriptor *(for the end of the school year)*
	Grade 3

Whole Number Concepts
 	
Students develop an understanding that numbers can represent a quantity. They also learn how to communicate with and about numbers. As students move to higher grades, they investigate relationships and patterns between numbers such as concepts of place value to be able to fluently add, subtract, multiply, and divide.
	For numbers up to 1000,
	• Compare and order numbers to make sense of quantities
	  ○ **Skip count** by 2, 3, 5, 10, 25, 100, using different starting points, to support the skills of repeated addition and learning multiples
	  ○ Backward **skip count** by 2, 5, and 10 to support the skill of repeated subtraction
	• Represent numbers in **concrete, pictorial, and symbolic forms**
	• Estimate the number of objects in a set up to 1000 by **decomposing** the set into smaller sets or by using **referents/benchmarks**
	  ○ Use benchmark numbers like multiples of 10, 50, and 100 to support estimation
	• Recognize if a number is odd or even and explain why (concept of pairs/dividing by 2: even numbers can be divided into 2 equal groups)
	• Understand **place value** as the relationship between the digits within a number and their value, to 999 (e.g., the digit 4 in 342 has the value of 40 ones or 4 tens) to make sense of quantities
	  ○ Demonstrate how numbers can be **decomposed** into 100s, 10s and 1s using place value (e.g., 140 is 14 tens OR 140 ones or 1 hundred, 4 tens and 0 ones, 47 is 4 tens and 7 ones OR 3 tens and 17 ones)
	  ○ Represent place value concepts in **concrete, pictorial, and symbolic forms** (e.g., use base-10 blocks to count 10 hundred squares, showing equality to 1 thousand cube)
	  ○ Demonstrate understanding of **expanded form** (e.g., 123 = 100 + 20 + 3)
	  ○ Understand the role of zero as a placeholder (e.g., 701 means that there are 0 tens)
	• Explore relationships between numbers based on place value (e.g., six 10s is 60, six 100s is 600)

Fractions and Decimals
 	
Fractions and decimals are numbers which can represent a part of a whole, part of a region, part of a set, or part of a length.
	• Represent fractions in **concrete, pictorial, and symbolic forms**
	  ○ As part of a whole (e.g., $\frac{1}{2}$ of a granola bar), part of a region (e.g., half the room is covered by carpet), part of a set (e.g., 2 out of 15 buttons are blue), or part of a length (e.g., 5/10 cm)
	  ○ Order and compare **unit fractions** (e.g., $\frac{1}{2} > \frac{1}{5}$ granola bar)

	○ Understand that the size of the fraction depends on the size of the whole (e.g., half a meter is longer than half a centimeter) • Identify and write fractions in **concrete** (e.g., measuring cups for baking), **pictorial** (e.g., colouring 2 out of 15 buttons blue), and **symbolic** (e.g., ½) forms

Skill - Computational Fluency (Operations)

Students who are proficient in Computational Fluency understand how operations change numbers. Students are introduced to, learn, and practice math facts (fluent and automatized knowledge of addition/subtraction and multiplication/division equations), and build fact fluency. Students compute fluently by applying known math facts and through the practice of computational and mental math strategies. Students understand how to apply these foundational skills to solve contextual problems.

Sub-skill	**Proficiency Descriptor** *(for the end of the school year)* **Grade 3**
Knowledge and Fluency of Math Facts ***Fact fluency*** *(accurate, efficient and flexible knowledge of math facts) based on conceptual understanding (e.g., whole number concepts, place value concepts, and representing math facts in* ***concrete, pictorial, and symbolic forms****) is encouraged rather than memorization of discrete facts. Rote memorization of facts should not be prioritized at this level.*	• Flexibly use a variety of **computational and mental math strategies** **(**e.g., **doubles**, **bridging to 10**, knowledge of **fact families**) to demonstrate addition and subtraction math fact fluency for numbers up to 20 • Use **concrete, pictorial, and symbolic forms** to represent math facts in a variety of activities such as games (e.g., card or dice games), discussions (e.g., a number talk of ways to make 17) and problem solving (connect to Algebraic Thinking)

Understanding of Operations
 	
Understanding of operations builds on math fact fluency. It also requires an understanding of concepts such as place value to be able to add, subtract, multiply, and divide larger numbers beyond known math facts. The development and practice of various computational and mental math strategies builds flexibility in doing operations and applying these skills to solve contextual problems.
	• Demonstrate understanding that addition brings sets of objects together and subtraction represents taking away from a set, or the difference between two amounts, for numbers up to 1000
	  ○ Use **computational and mental math strategies** such as **decomposing** and **recomposing** (regrouping) by place value and **compensating**
	  ○ Represent addition and subtraction using **concrete, pictorial, and symbolic forms** (e.g., base-10 blocks, number line, hundred chart)
	  ○ Estimate sums and differences to 1000
	• Solve **contextual** addition and subtraction **problems** which may require more than one step
	  ○ Choose the appropriate operation and strategy to solve a **contextual problem**
	  ○ Explain and justify their strategy and solution to others using mathematical language (e.g., "To solve 954 – 496 I bridged to 500 by adding 4, then I counted by hundreds to 900, then added 54. The answer is 458.")
	• **Compose and decompose** groups or arrays of objects using **concrete, pictorial, and symbolic forms** to demonstrate understanding that multiplication is the repeated addition of groups of the same size, and division is repeated subtraction of groups of the same size or splitting a group into equal sized shares
	  ○ Demonstrate understanding that multiplication and division are related/opposite operations (e.g., show **fact families** of conceptual relationships such as 2 groups of 7 objects is 14 in total, and 14 can be split into 2 equal groups of 7; relate forward skip counting to multiples)

Skill - Patterning and Algebraic Thinking

Students develop proficient Algebraic Thinking through recognizing patterns and trends, identifying and representing relationships between numbers, making generalizations, and analyzing change.

Sub-skill	Proficiency Descriptor *(for the end of the school year)* Grade 3

Patterning
 	
Noticing relationships in patterns such as repetition helps students develop skills to observe, identify, and classify, and supports developing prediction skills.
	• Represent increasing or decreasing **number patterns** in **concrete, pictorial, and symbolic forms** (e.g., using blocks, natural objects; numbers; music, movements)
	  ○ Describe and represent **pattern rules** in symbolic forms, (e.g., blocks of 1, 2, 4, 8 … is doubling the number of blocks starting from 1)
	  ○ Predict elements of, extend a pattern, and explain their reasoning
	• Explore more complex repeating patterns (e.g., with more than 3 elements, spatial patterns, circular patterns, patterns in which 2 or more attributes change such as colour and shape)
	• Explore increasing and decreasing **number patterns** such as the Fibonacci sequence

Algebraic Thinking
 	
Students explore concepts and symbols of equality and inequality. They apply understanding of these ideas to solve for unknown values and understand relationships between numbers.
	• Use **concrete, pictorial, and symbolic representations** to solve one-step addition and subtraction equations of math facts up to 20 (connection to Operations), with an unknown number e.g.:
	  ○ Start unknown (e.g., n + 15 = 20 or n + 4 = 9)
	  ○ Change unknown (e.g., 12 + n = 20 or 2 + n = 8)
	  ○ Result unknown (e.g., 6 + 13 = n or 9 + 3 = n)
	  ○ Recognize implicit start unknown, change unknown, or result unknown situations in numeracy problems. Translate these equation types to a **concrete, pictorial, and/or symbolic form**
	  ○ Apply knowledge of **fact families** of addition and subtraction to help solve equations

Skill - Geometry and Measurement (Spatial Sense)

Students develop strong Spatial Sense in part through the study of Geometry and Measurement and can demonstrate the ability to describe objects and position with mathematical language and numerical measurement.

Sub-skill	Proficiency Descriptor *(for the end of the school year)*
	Grade 3

2D Shapes and 3D Objects
 	
Noticing attributes of shapes helps students develop skills to observe, identify, classify, and supports creativity and design.
	• Identify, describe using mathematical language, and sort **3D objects** using more than one **attribute**, and explain their thinking
	  ○ Identify and describe defining attributes of 3D design elements from various cultures, including local First Nations
	  ○ Identify 3D objects according to the number of edges and vertices
	  ○ Compare and contrast 3D **skeletons** and **nets**
	• Use mathematical language to demonstrate understanding of **preservation of shape**
	• Identify and describe 3D objects in the real world (e.g., bentwood box, ice cream cone, longhouses, house poles, soccer ball, boxes, cans, dice)

Measurement
 	
Measurements iteratively use standard units (from established systems such as the metric system) and/or non-standard units (e.g., using hands or blocks to measure height). Mathematical language to describe position can be descriptive (qualitative) or numerical (quantitative). This sub-skill includes concepts of time.
	• Use **standard** metric units (e.g., millimeter) to accurately measure and record the length, width, or height of an object
	• Understand the relationship between metric units of length (e.g., 100 cm = 1 m)
	• **Estimate** the length, height, width, mass, and capacity of an object, using quantitative mathematical language (e.g., centimeters, milliliters) or by using **referents** (e.g., this cup holds 100 milliliters, this jug holds about 10 times more)
	• Understand units of time (e.g., second, minute, hour, day, week, month, year)
	  ○ Identify properties of time-keeping devices (e.g., analog clocks have 2-3 hands, how digital clocks look, how stop-watches work, lay-out of calendars)
	  ○ Telling time is not expected at this level
	• Identify important relationships-between units of time
	  ○ 1 minute = 60 seconds; 1 hour = 60 minutes; 1 day = 24 hours; 1 week = 7 days; 365 days = 1 year = 12 months
	  ○ State the months of the year, in order
	  ○ Estimate time using environmental references (e.g., natural daily/seasonal cycles, weather patterns)
	• Describe similarities and differences between different calendar and time keeping systems or references from around the world, including from local First Nations

Skill - Data and Probability (Data Literacy)

By developing proficient Data Literacy skills, students are able to demonstrate the ability to understand, analyze, and communicate data, and the ability to predict the likelihood of an event. As students move to later grades, they will use their foundational mathematics skills to collect, communicate, and discuss data across all areas of the curriculum.

Sub-skill	Proficiency Descriptor *(for the end of the school year)* Grade 3

Understanding, Analyzing, and Communicating Data
 	
Graphs help to visually represent observations and data. Students build proficiency in collecting data to communicate in various types of graphs and inferring information from graphs.
	• Collect and communicate data using graphs
	  ○ Record data using a chosen method (e.g., tally marks, numerals, or counters)
	  ○ Choose and use a suitable graph or visual to communicate the data (e.g., **bar graphs**, **pictographs**)
	  ○ Represent information on a graph using **one-to-one correspondence**
	• Describe, compare, and discuss data
	• Interpret and discuss graphs by making quantitative comparisons (e.g., 4 people like broccoli, 7 people like brussels sprouts, and 10 people like carrots best. Therefore our class's favourite vegetable is carrots)

Probability *Students discuss the likelihood of an event using descriptive language of probability. As students move to later grades, they will begin to use quantitative descriptors such as describing probability with fractions, decimals, and percentages.*	• Describe the likelihood of simulated events (such as a coin toss), using comparative language related to probability (e.g., certain, uncertain; more, less, or equally likely) and explain their thinking • Demonstrate an understanding of chance through inquiry ○ Conduct probability experiments such as tossing a coin, drawing from a bag, using spinners, and rolling dice • Describe results using comparative language (e.g. equal probability to flip heads or tails on a coin)

Skill - Financial Literacy

Students with a strong understanding of Financial Literacy will develop the tools required to practice sound financial decision making in the future. This skill provides real world context for demonstrating learning in Number Sense and Computational Fluency.

Sub-skill	Proficiency Descriptor *(for the end of the school year)*
	Grade 3
Currency *Identifying, understanding the value of, and combining coins and bills fluently, with an emphasis on Canadian currency.*	• Identify, name, and describe Canadian bills and coins by their size, design, and value • Determine the value of mixed combinations of bills and coins (using whole dollars, or cents rather than decimal notation) (connection to Operations) • Solve 3-digit addition and subtraction questions (using whole dollars, or cents rather than decimal notation, e.g., 125¢ + 110¢ = 235¢) (connection to Computational Fluency) ○ Using a variety of **concrete** (play money), **pictorial, or symbolic forms** • Explain their thinking process (e.g. describe the coins and bills to be used to pay)

Financial Planning and Decision Making
 	
Concepts of earning, saving, spending, and making financial plans and decisions.
	• Understand and explore concepts such as
	  ○ Payments can be made in flexible ways (e.g., cash, cheques, credit, electronic transactions, trading goods and services)
	  ○ Different developmentally and contextually appropriate ways of earning money to reach a financial goal (e.g., recycling, holding bake sales, selling items, walking a neighbour's dog) (Core Competencies)
	  ○ Trading and forms of currency in First Peoples history (Social Studies)

Definitions

Note: This is a complete list of foundational skill definitions from Kindergarten through Grade 4. Many of these terms are found in multiple Skills/Sub-skills and across grades. Therefore, the example included may or may not be appropriate for the grade.

2D shapes: includes but not limited to circle, square, rectangle, triangle, heart, kite
3D objects: includes but not limited to sphere, cone, cube, rectangular prism, triangular pyramid and square pyramid
Attribute: description of an element in a pattern, (e.g., colour, shape, size, number/letter/symbol, object, direction, position)
Bar graph: A way of showing numerical data by category using the height or length of a bar. Also known as a column chart. Bar graphs are used to compare things between different groups or to show data collected in different times or places.
Benchmarks: a familiar, measurable quantity used to help estimate or understand other quantities, (e.g., 5, 10, 100, 1000, 25, 50)
Cardinality: knowing that the last number said when counting represents the quantity of objects in a set
Change tasks: a task in which a student demonstrates changing a quantity using concrete, pictorial, and symbolic models (e.g., using blocks to show changing 8 to 12 by adding 4 more blocks)
Compose/Recompose: building or rebuilding a set of objects or a number from smaller parts. For example, 5 can be made from 1 and 4 or 2 and 3
Concrete forms: concrete materials or physical objects help students learn through actions such as placing, moving, grouping, or splitting objects. Examples include manipulatives (e.g., base-10 blocks, counters), natural or found materials, loose parts
Concrete graph: A graph that organizes and displays data using concrete objects like blocks or stickers, with appropriate labels.
Conservation: knowing that the number of objects remains the same despite changing the size of objects or how they are laid out
Contextual problem: a problem that is set within a real-world or practical situation, requiring students to apply mathematical concepts to solve it. Students identify and apply the best mathematical operation and strategy to fit the context, and based on their current knowledge and available tools
Core: repeated element in a repeating pattern, (e.g., AAB is the core of AABAABAAB)
Decompose: breaking down a number into smaller parts. For example, 12 = 10 + 2, or 147 can be broken into 100, 40, and 7 OR 14 tens and 7 ones
Equivalents: quantities that are equal in value, function, amount, or meaning, but not necessarily number, i.e., 1 m = 100 cm, 4 quarters = 1 loonie
Estimate: to approximate a number, calculation, quantity, or measurement based on an educated guess, rounding, or a visual comparison with a referent or benchmark
Expanded form: breaking down numbers as the sum of the digits and their place value, (e.g., 123 = 100 + 20 + 3)
Fact families: sets of related addition/subtraction or multiplication/division math facts such as 12 + 6 = 18 and 18 – 6 = 12, or 2 x 7 = 14 and 14 ÷ 7 = 2. Fact families illustrate how sets can be decomposed and recomposed.
Fractions: numbers which can represent a part of a whole, part of a region, part of a set, or part of a length. The **denominators** of fractions represent equal-sized portions of a whole or unit. The **numerators** represent the number of portions within the fraction. For example, $\frac{2}{3}$ represents 2 portions (the numerator) out of a whole that has been divided into 3 portions (the denominator).
Irregular polygons: 2D shapes in which all sides are not equal in length or all angles not equal in measure
Many-to-one correspondence: On a concrete graph or pictograph, one symbol represents a group of objects (e.g., one square may represent five cookies)
Math facts: fluent and automatized knowledge of addition/subtraction and multiplication/division equations **One-to-one correspondence**: when counting the number of objects in a set, using one number per object for accuracy
Nets: representations of 3D objects if cut open and laid flat
Non-standard units: measurement units using everyday objects (e.g., a pencil, arm, shoe)
Non-uniform units: not consistent in size (e.g., children's hands, pencils)
Number pattern: also known as number sequences, are a series of numbers that follow a specific rule or have a relationship between successive numbers. Number sequences can be arithmetic (adding or subtracting by a constant, aka increasing or decreasing patterns. (e.g., 75, 70, 65, 60, 55, ...), or geometric (multiplying or dividing by a constant e.g., 1, 2, 4, 8, 16...)

One to one correspondence: On a concrete graph or pictograph, one picture symbol corresponds to one unit of data (e.g., one dot represents one flower)
Pattern rule: Pattern rules describe the relationship between elements in the pattern. Pattern rules can be based on attributes or mathematical operations. Patterns can be repeating, or number (increasing or decreasing) patterns.
Perimeter: total distance around the outside of a 2D shape. The perimeter of a circle is also called the circumference
Pictograph: A way of showing data using images, where each image stands for quantity (one to one correspondence or many to one correspondence). Pictographs are used to express large amounts of information in a simple manner as it is easy to read.
Pictorial forms: pictorial forms help students see a model which represents a number. Examples include pictures of counting collections, number lines, tally marks, 10-frames
Place value: the relationship between the digits within a number and their value, (e.g., the digit 4 in 49 has the value of 40)
Polygons: an enclosed 2D shape made up of straight lines
Preservation of shape: the orientation/position of a shape will not change its attributes but will change its appearance
Ratio table: a table to help understand the relationship between the quantities (e.g., a t-chart could record the total number of meals eaten at a camp, per day—one column could be the number of days, another column could be the number of meals). Ratio tables are also known as t-tables, function machines, or in and out machines
Referents: a known number of concrete or pictorial representations that can be used for comparison, to help estimate an unknown quantity. For example, a 10-frame (grid consisting of 2 lines of 5 squares) can be used as a referent for amounts less than or more than 5
Regular polygons: 2D shapes in which all sides are equal in length and all angles equal in measure
Skeletons: representation of a 3D shape without the faces
Skip counting: method of counting in which students add a number to the previous number (also known as multiples). For example, skip counting by 5, starting at 0 is 0, 5, 10, 15, …
Stable order counting: accurately counting numbers in the correct sequence
Standard units: measurement units within established systems such as the metric system **Subitize**: instantly recognizing how many there are in a set without counting, e.g. knowing the number of dots on dice without counting. Subitizing can usually be done for a group of up to 5 objects
Symbolic forms: symbolic forms are abstract mathematical notations which represent numbers. Examples include numerals, tallies, musical notes, fractions
Symbols of equality and inequality: The = sign means "the same as", e.g., 4 + 6 = 3 + 7. Elements on both sides of the = sign are balanced regardless of size or shape. Symbols of inequality include less than < ; greater than > ; and not equal ≠
Uniform units: consistent in size (e.g., interlocking cubes, standard paper clips)
Unit fractions: fractions with 1 in the numerator, such as $\frac{1}{2}$ or $\frac{1}{5}$. They allow the comparison of the size of fractions (e.g., $\frac{1}{2}$ granola bar > $\frac{1}{5}$ granola bar)

Computational and mental math strategies:
Addition and subtraction strategies

- **Compensating:** bridging to or decomposing to a benchmark or friendly number [ex: 47 + 28 → add 3 to 47 to make 50 (friendly number) → subtract 3 from 28 to compensate = 25 → 50 + 25 = 75]
- **Commutative property:** the order of the numbers in an addition equation does not change the sum, (e.g., 3 + 5 = 5 + 3 = 8)
- **Counting on and counting back:** starting from a number and counting on by the added number or back by the subtracted number
- **Decomposing:** breaking down a number into smaller or simpler parts to support computation. [ex: 47 + 28 → decompose by place value: 47 = 40 + 7; 28 = 20 + 8 → add by place value: 40 + 20 = 60; 7 + 8 = 15 → add total: 60 + 15 = 75]
- **Doubles:** e.g., 3 + 3 = 6, 7 + 7 = 14
- **Friendly numbers**: bridging to or decomposing to a benchmark or friendly number (ex. 7 = 5 + 2, 5 is a "friendly number"). Often used in conjunction compensating, decomposing, or making 10/bridging over 10 strategies

- **Hundred Chart:** jumping along the hundred chart (similar to counting up and counting back) [ex: 47 + 28 → Start at 47, add 20 (move 2 rows down) to 67, then add 8 (move 8 squares to the right) to 75]
- **Making 10/bridging over 10**: "bridging" to the nearest 10 (ex. 8 + 5. The 8 can be "bridged" to 10 by adding 2 out of 5. Then, add the remaining 3 → 8 + 5 = 8 + 2 + 3 = 10 + 3 = 13)
- **Open number line**: jumping along the number line (similar to counting up and counting back) [ex: 47 + 28 → start at 47, jump 20 to 67, jump 3 to 70, jump 5 to 75]
- **Skip counting:** method of counting in which students add a number to the previous number. For example, skip counting by 5, starting at 0 is 0, 5, 10, 15, …

Multiplication and division strategies: The strategies below utilize an understanding of place value and are encouraged. Traditional methods (e.g., long division) can rely on memorization of a procedure and may not effectively demonstrate a true understanding that multiplication is the addition of multiple groups (e.g., 12 x 2 is 2 groups of 10 and 2 groups of 2), and division is splitting a group into equal sized shares (e.g., 36 ÷ 3 can be shown as 30 ÷ 3 shares and 6 ÷ 3 shares)

- **Decomposing:** breaking down a number into smaller or simpler parts to support computation (related to distributive property)
- **Distributive property (partial products):** a way to break down multiplication into smaller steps. Example: 5 x 23 → decompose 23 into 20 and 3 → 5 x 23 = 5 x 20 + 5 x 3 = 115