Area of Learning: MATHEMATICS — Pre-calculus

BIG IDEAS

Using **inverses** is the foundation of solving equations and can be extended to relationships between functions.

Understanding the characteristics of families of **functions** allows us to model and understand relationships and to build connections between classes of functions.

**Transformations** of shapes extend to functions and relations in all of their representations.

Learning Standards

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<th>Curricular Competencies</th>
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<td><strong>Students are expected to do the following:</strong></td>
<td><strong>Students are expected to know the following:</strong></td>
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| **Reasoning and modelling** | • transformations of functions and relations  
• exponential functions and equations  
• geometric sequences and series  
• logarithms: operations, functions, and equations  
• polynomial functions and equations  
• rational functions  
• trigonometry: functions, equations, and identities |
| • Develop **thinking strategies** to solve puzzles and play games  
• Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools**  
• **Estimate reasonably** and demonstrate **fluent, flexible, and strategic** thinking about number  
• **Model** with mathematics in **situational contexts**  
• **Think creatively** and with **curiosity and wonder** when exploring problems | |
| **Understanding and solving** | |
| • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving  
• **Visualize** to explore and illustrate mathematical concepts and relationships  
• Apply **flexible and strategic approaches** to **solve problems**  
• Solve problems with **persistence and a positive disposition**  
• Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures | |
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<th>Curricular Competencies</th>
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<td><strong>Communicating and representing</strong></td>
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<tr>
<td>• Explain and justify mathematical ideas and decisions in many ways</td>
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<td>• Represent mathematical ideas in concrete, pictorial, and symbolic forms</td>
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<td>• Use mathematical vocabulary and language to contribute to discussions in the classroom</td>
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<td>• Take risks when offering ideas in classroom discourse</td>
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<td><strong>Connecting and reflecting</strong></td>
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<tr>
<td>• Reflect on mathematical thinking</td>
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<td>• Connect mathematical concepts with each other, other areas, and personal interests</td>
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<td>• Use mistakes as opportunities to advance learning</td>
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<td>• Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts</td>
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• Inverses:
  – *undo* the operations within an expression or function to reduce the expression to an identity (e.g., \( x = \) ).

*Sample questions to support inquiry with students:*
  – How can the inverse help to solve an equation?
  – How is solving an equation related to identifying the specific input for a function, given a specific output?
  – How are exponential and logarithmic functions related?
  – How are the laws of exponents connected to the laws of logarithms?
  – What are some other examples of inversely related functions?
  – How are inverses related graphically, and why?
  – How is solving an exponential equation similar to solving a trigonometric equation?
  – How are inverse operations related to solving a polynomial equation by factoring?
  – What is the value of using trigonometric identities to find equivalent expressions?
  – Why do some equations have extraneous roots and other equations do not?

• Functions:

*Sample questions to support inquiry with students:*
  – How do we decide which kind of function to use to model a given problem?
  – What do functions and relations look like beyond the visible axes?
  – A set of data looks like a parabola, but it is not. What function could be used to model this data?
  – What does the number of zeros tell us about a function?
  – What connections do we see within the characteristics of a particular class of function?

• Transformations:

*Sample questions to support inquiry with students:*
  – How can we tell whether a transformation will have invariant points?
  – Under what circumstances will different transformations produce the same result?
  – How do graphical transformations affect the tables of values?
  – How does a transformation affect a point found at the origin as compared to a point on an axis or a point in one of the four quadrants?
  – How can a rational function of the form \( y = \frac{ax+b}{cx+d} \) be considered as a transformation of the reciprocal function \( y = \frac{1}{x} \)?
Curricular Competencies – Elaborations

- **thinking strategies:**
  - using reason to determine winning strategies
  - generalizing and extending

- **analyze:**
  - examine the structure of and connections between mathematical ideas (e.g., exponential functions to geometric sequences)

- **reason:**
  - inductive and deductive reasoning
  - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)

- **technology:**
  - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
  - can be used to for a wide variety of purposes, including:
    - exploring and demonstrating mathematical relationships
    - organizing and displaying data
    - generating and testing inductive conjectures
    - mathematical modelling

- **other tools:**
  - manipulatives such as algebra tiles and other concrete materials

- **Estimate reasonably:**
  - be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., the zeros of a graphed polynomial function)

- **fluently, flexible and strategic thinking:**
  - includes:
    - using known facts and benchmarks, partitioning, applying whole number strategies to rational numbers and algebraic expressions
    - choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?)

- **Model:**
  - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
  - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it

- **situational contexts:**
  - including real-life scenarios and open-ended challenges that connect mathematics with everyday life

- **Think creatively:**
  - by being open to trying different strategies
  - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music
Curricular Competencies – Elaborations

- curiosity and wonder:
  - asking questions to further understanding or to open other avenues of investigation
- inquiry:
  - includes structured, guided, and open inquiry
  - noticing and wondering
  - determining what is needed to make sense of and solve problems
- Visualize:
  - create and use mental images to support understanding
  - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- flexible and strategic approaches:
  - deciding which mathematical tools to use to solve a problem
  - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- solve problems:
  - interpret a situation to identify a problem
  - apply mathematics to solve the problem
  - analyze and evaluate the solution in terms of the initial context
  - repeat this cycle until a solution makes sense
- persistence and a positive disposition:
  - not giving up when facing a challenge
  - problem solving with vigour and determination
- connected:
  - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
  - by posing and solving problems or asking questions about place, stories, and cultural practices
- Explain and justify:
  - use mathematical arguments to convince
  - includes anticipating consequences
- decisions:
  - Have students explore which of two scenarios they would choose and then defend their choice.
- many ways:
  - including oral, written, visual, use of technology
Curricular Competencies – Elaborations

• Represent:
  – using models, tables, graphs, words, numbers, symbols
  – connecting meanings among various representations

• discussions:
  – partner talks, small-group discussions, teacher-student conferences

• discourse:
  – is valuable for deepening understanding of concepts
  – can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions

• Reflect:
  – share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions

• Connect mathematical concepts:
  – to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)

• mistakes:
  – range from calculation errors to misconceptions

• opportunities to advance learning:
  – by:
    ▪ analyzing errors to discover misunderstandings
    ▪ making adjustments in further attempts
    ▪ identifying not only mistakes but also parts of a solution that are correct

• Incorporate:
  – by:
    ▪ collaborating with Elders and knowledge keepers among local First Peoples
    ▪ exploring the First Peoples Principles of Learning (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
    ▪ making explicit connections with learning mathematics
    ▪ exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections

• knowledge:
  – local knowledge and cultural practices that are appropriate to share and that are non-appropriated

• practices:
  – Bishop’s cultural practices: counting, measuring, locating, designing, playing, explaining
  – Aboriginal Education Resources
  – Teaching Mathematics in a First Nations Context, FNESC
• transformations:
  – of graphs and equations of parent functions and relations (e.g., absolute value, radical, reciprocal, conics, exponential, logarithmic, trigonometric)
  – vertical and horizontal translations, stretches, and reflections
  – inverses: graphs and equations
  – extension:
    ▪ recognizing composed functions (e.g., $y =$)
    ▪ operations on functions
• exponential:
  – graphing, including transformations
  – solving equations with same base and with different bases, including base $e$
  – solving problems in situational contexts
• geometric:
  – common ratio, first term, general term
  – geometric sequences connecting to exponential functions
  – infinite geometric series
  – sigma notation
• logarithms:
  – applying laws of logarithms
  – evaluating with different bases
  – using common and natural logarithms
  – exploring inverse of exponential
  – graphing, including transformations
  – solving equations with same base and with different bases
  – solving problems in situational contexts
• polynomial:
  – factoring, including the factor theorem and the remainder theorem
  – graphing and the characteristics of a graph (e.g., degree, extrema, zeros, end-behaviour)
  – solving equations algebraically and graphically
• rational:
  – characteristics of graphs, including asymptotes, intercepts, point discontinuities, domain, end-behaviour
### Content – Elaborations

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- **trigonometry:**
  - examining angles in standard position in both radians and degrees
  - exploring unit circle, reference and coterminal angles, special angles
  - graphing primary trigonometric functions, including transformations and characteristics
  - solving first- and second-degree equations (over restricted domains and all real numbers)
  - solving problems in situational contexts
  - using identities to reduce complexity in expressions and solve equations (e.g., Pythagorean, quotient, double angle, reciprocal, sum and difference)