

BIG IDEAS

Diagrams are fundamental to investigating, communicating, and discovering properties and relations in geometry.

Finding **invariance amidst variation** drives geometric investigation.

Geometry involves creating, testing, and refining **definitions**.

The **proving process** begins with conjecturing, looking for counter-examples, and refining the conjecture, and the process may end with a written proof.

Geometry stories and applications vary across cultures and time.

Learning Standards

Curricular Competencies	Content
<p><i>Students are expected to do the following:</i></p> <p>Reasoning and modelling</p> <ul style="list-style-type: none"> • Develop thinking strategies to solve puzzles and play games • Engage in spatial reasoning in a dynamic environment • Explore, analyze, and apply mathematical ideas using reason, technology, and other tools • Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about number • Model with mathematics in situational contexts • Think creatively and with curiosity and wonder when exploring problems <p>Understanding and solving</p> <ul style="list-style-type: none"> • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, inquiry, and problem solving • Visualize to explore and illustrate geometric concepts and relationships • Apply flexible and strategic approaches to solve problems • Solve problems with persistence and a positive disposition • Engage in problem-solving experiences connected with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures 	<p><i>Students are expected to know the following:</i></p> <ul style="list-style-type: none"> • geometric constructions • parallel and perpendicular lines: <ul style="list-style-type: none"> – circles as tools in constructions – perpendicular bisector • circle geometry • constructing tangents • transformations of 2D shapes: <ul style="list-style-type: none"> – isometries – non-isometric transformations • non-Euclidean geometries

Learning Standards (continued)

Curricular Competencies	Content
<p>Communicating and representing</p> <ul style="list-style-type: none"> • Explain, justify, and evaluate geometric ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use geometric vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse <p>Connecting and reflecting</p> <ul style="list-style-type: none"> • Reflect on geometric thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with mathematical concepts 	

Big Ideas – Elaborations	MATHEMATICS – Geometry Grade 12
<ul style="list-style-type: none"> • Diagrams: <i>Sample questions to support inquiry with students:</i> <ul style="list-style-type: none"> – How would we describe a specific geometric object to someone who cannot see it? – What properties can we infer from a diagram? – What behaviours can we infer from a dynamic diagram? 	

Big Ideas – Elaborations

• **invariance amidst variation:**

- Invariance amidst variation can be more easily experienced using current technology and dynamic diagrams. For example, the sum of the angles in planar triangles is invariant no matter what forms a triangle takes.

Sample questions to support inquiry with students:

- How do we construct geometric shapes that maintain properties under variation?
- What properties change and stay the same when we vary a square, parallelogram, triangle, and so on?
- How can the Pythagorean theorem be restated in terms of variance and invariance?

• **definitions:**

- are seldom the starting point in geometry

Sample questions to support inquiry with students:

- How does variation help to refine our definitions of shapes?
- How would we define a square (or a circle) in different ways? When would one definition be better to work with than another?
- How can the definition of a shape be used in constructing the shape?
- How can we modify a definition of a shape to define a new shape?

• **proving process:**

Sample questions to support inquiry with students:

- Can we make a conjecture about the diagonals of a polygon? Can we find a counter-example to our conjecture?
- How can one conjecture about a *specific* shape lead to making another more *general* conjecture about a family of shapes?
- How can we be sure that a proof is complete?
- Can we find a counter-example to a conjecture?
- How can different proofs bring out different understandings of a relationship?

• **Geometry:**

- Geometry is more than a list of axioms and deductions. Non-Western and modern geometry is concerned with shape and space and is not always axiomatic. It is not always about producing a theorem; rather, it is about modelling mathematical and non-mathematical phenomena using geometric objects and relations. Today geometry is used in a multitude of disciplines, including animation, architecture, biology, carpentry, chemistry, medical imaging, and art.

Sample questions to support inquiry with students:

- Can we find geometric relationships in local First Peoples art or culture?
- Can we make geometric connections to story, language, or past experiences?
- What do we notice about and how would we construct common shapes found in local First Peoples art?
- How has the notion of “proof” changed over time and in different cultures?
- How are geometric ideas implemented in modern professions?

Curricular Competencies – Elaborations

- **thinking strategies:**
 - using reason to determine winning strategies
 - generalizing and extending
- **spatial reasoning:**
 - being able to think about shapes (real or imagined) and mentally transform them to notice relationships
- **analyze:**
 - examine the structure of and connections between geometric ideas (e.g., parallel and perpendicular lines, circle geometry, constructing tangents, transformations)
- **reason:**
 - inductive and deductive reasoning
 - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
- **technology:**
 - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
 - can be used for a wide variety of purposes, including:
 - exploring and demonstrating geometrical relationships
 - organizing and displaying data
 - generating and testing inductive conjectures
 - mathematical modelling
- **other tools:**
 - paper and scissors, straightedge and compass, ruler, and other concrete materials
- **Estimate reasonably:**
 - be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., congruencies, angles, lengths)
- **fluent, flexible, and strategic thinking:**
 - being able to generate a family of shapes and apply characteristics across the family
- **Model:**
 - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
 - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
- **situational contexts:**
 - including real-life scenarios and open-ended challenges that connect mathematics with everyday life
- **Think creatively:**
 - by being open to trying different strategies
 - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music

Curricular Competencies – Elaborations

- **curiosity and wonder:**
 - asking questions to further understanding or to open other avenues of investigation
- **inquiry:**
 - includes structured, guided, and open inquiry
 - noticing and wondering
 - determining what is needed to make sense of and solve problems
- **Visualize:**
 - create and use mental images to support understanding
 - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- **flexible and strategic approaches:**
 - deciding which mathematical tools to use to solve a problem
 - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- **solve problems:**
 - interpret a situation to identify a problem
 - apply mathematics to solve the problem
 - analyze and evaluate the solution in terms of the initial context
 - repeat this cycle until a solution makes sense
- **persistence and a positive disposition:**
 - not giving up when facing a challenge
 - problem solving with vigour and determination
- **connected:**
 - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
 - by posing and solving problems or asking questions about place, stories, and cultural practices
- **Explain and justify:**
 - use geometrical arguments to convince
 - includes anticipating consequences
- **decisions:**
 - Have students explore which of two scenarios they would choose and then defend their choice.
- **many ways:**
 - including oral, written, visual, gestures use of technology
 - communicating effectively according to what is being communicated and to whom

Curricular Competencies – Elaborations

- **Represent:**
 - concretely, diagrammatically, symbolically, including using models, tables, graphs, words, numbers, symbols
- **discussions:**
 - partner talks, small-group discussions, teacher-student conferences
- **discourse:**
 - is valuable for deepening understanding of concepts
 - can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions
- **Reflect**
 - share the geometric thinking of self and others, including evaluating strategies and solutions, finding counter-examples, extending, posing new problems and questions, proving results
- **Connect mathematical concepts:**
 - to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
- **mistakes:**
 - range from calculation errors to misconceptions
- **opportunities to advance learning:**
 - by:
 - analyzing errors to discover misunderstandings
 - making adjustments in further attempts
 - identifying not only mistakes but also parts of a solution that are correct
- **Incorporate:**
 - by:
 - collaborating with Elders and knowledge keepers among local First Peoples
 - exploring the [First Peoples Principles of Learning](#) (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
 - making explicit connections with learning mathematics
 - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
- **knowledge:**
 - local knowledge and cultural practices that are appropriate to share and that are non-appropriated
- **practices:**
 - [Bishop’s cultural practices](#): counting, measuring, locating, designing, playing, explaining
 - [Aboriginal Education Resources](#)
 - [Teaching Mathematics in a First Nations Context](#), FNECS

Content – Elaborations

- **constructions:**
 - angles, triangles, triangle centres, quadrilaterals
- **parallel and perpendicular:**
 - angle bisector
- **circles as tools:**
 - constructing equal segments, midpoints
- **circle geometry:**
 - properties of chords, angles, and tangents to mobilize the proving process
- **constructing tangents:**
 - lines tangent to circles, circles tangent to circles, circles tangent to three objects (e.g., points [PPP], three lines [LLL])
- **isometries:**
 - transformations that maintain congruence (translations, rotations, reflections)
 - composition of isometries
 - tessellations
- **non-isometric transformations:**
 - dilations and shear
 - topology
- **non-Euclidean geometries:**
 - perspective, spherical, Taxicab, hyperbolic
 - tessellations