**Area of Learning: MATHEMATICS — Calculus Grade 12**

**BIG IDEAS**

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| The **concept of a limit** is foundational to calculus. |  | Differential calculus develops the concept of **instantaneous rate of change**. |  | Integral calculus develops the concept of determining a product involving a **continuously changing** quantity over an interval. |  | Derivatives and integrals are **inversely related**. |

**Learning Standards**

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| **Curricular Competencies** | **Content** |
| *Students are expected to do the following:*  Reasoning and modelling   * Develop **thinking strategies** to solve puzzles and play games * Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** * **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number * **Model** with mathematics in **situational contexts** * **Think creatively** and with **curiosity and wonder** when exploring problems   Understanding and solving   * Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving * **Visualize** to explore and illustrate mathematical concepts and relationships * Apply **flexible and strategic approaches** to **solve problems** * Solve problems with **persistence and a positive disposition** * Engage in problem-solvingexperiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures | *Students are expected to know the following:*   * **functions** and graphs * **limits:**   + left and right limits   + limits to infinity   + continuity * **differentiation:**   + **rate of change**   + **differentiation rules**   + higher order, implicit   + **applications** * **integration:**   + **approximations**   + fundamental theorem of calculus   + **methods of integration**   + **applications** |

**Area of Learning: MATHEMATICS — Calculus Grade 12**

**Learning Standards (continued)**

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| **Curricular Competencies** | **Content** |
| Communicating and representing   * **Explain and justify** mathematical ideas and **decisions** in **many ways** * **Represent** mathematical ideas in concrete, pictorial, and symbolic forms * Use mathematical vocabulary and language to contribute to **discussions** in the classroom * Take riskswhen offering ideas in classroom **discourse**   Connecting and reflecting   * **Reflect** on mathematical thinking * **Connect mathematical concepts** with each other, other areas,  and personal interests * Use **mistakes** as **opportunities to advance learning** * **Incorporate** First Peoples worldviews, perspectives, **knowledge**,  and **practices** to make connections with computer science concepts |  |

| **MATHEMATICS – Calculus  Big Ideas – Elaborations Grade 12** |
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| * **concept of a limit:**   + Differentiation and integration are defined using limits.   *Sample questions to support inquiry with students:*   * + Why is a limit useful?   + How can we use historical examples (e.g., Achilles and the tortoise) to describe a limit? * **instantaneous rate of change:**   + developing rate of change from average to instantaneous   *Sample questions to support inquiry with students:*   * + How can a rate of change be instantaneous?   + When do we use rate of change? * **continuously changing:**   + area (height x width) under a curve where the height of the region is changing; volume of a solid (area x length) where cross-sectional area is changing; work (force x distance) where force is changing   + Finding these products requires finding an infinite sum.   *Sample questions to support inquiry with students:*   * + What is the value of using rectangles to approximate the area under a curve?   + Why is the fundamental theorem of calculus so fundamental? * **inversely related:**   + The fundamental theorem of calculus describes the relationship between integrals and antiderivatives.   *Sample questions to support inquiry with students:*   * + How are derivatives and integrals related?   + Why are antiderivatives important?   + What is the difference between an antiderivative and an integral? |

| **MATHEMATICS – Calculus  Curricular Competencies – Elaborations Grade 12** |
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| * **thinking strategies:**   + using reason to determine winning strategies   + generalizing and extending * **analyze:**   + examine the structure of and connections between mathematical ideas (e.g., limits, derivatives, integrals) * **reason:**   + inductive and deductivereasoning   + predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding) * **technology:**   + graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps   + can be used for a wide variety of purposes, including:     - exploring and demonstrating mathematical relationships     - organizing and displaying data     - generating and testing inductive conjectures     - mathematical modelling * **other tools:**   + manipulatives such as algebra tiles and other concrete materials * **Estimate reasonably:**   + be able to defend the reasonableness of an estimate across mathematical contexts * **fluent, flexible, and strategic thinking:**   + includes:     - using known facts and benchmarks, partitioning, applying number strategies to approximate limits, derivatives, and integrals     - choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?) * **Model:**   + use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)   + take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it * **situational contexts:**    + including real-life scenarios and open-ended challenges that connect mathematics with everyday life * **Think creatively:**   + by being open to trying different strategies   + refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music * **curiosity and wonder:**   + asking questions to further understanding or to open other avenues of investigation * **inquiry:**   + includes structured, guided, and open inquiry   + noticing and wondering   + determining what is needed to make sense of and solve problems * **Visualize:**    + create and use mental images to support understanding   + Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings,  and diagrams. * **flexible and strategic approaches:**   + deciding which mathematical tools to use to solve a problem   + choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play) * **solve problems:**   + interpret a situation to identify a problem   + apply mathematics to solve the problem   + analyze and evaluate the solution in terms of the initial context   + repeat this cycle until a solution makes sense * **persistence and a positive disposition:**   + not giving up when facing a challenge   + problem solving with vigour and determination * **connected:**   + through daily activities, local and traditional practices, popular media and news events, cross-curricular integration   + by posing and solving problems or asking questions about place, stories, and cultural practices * **Explain and justify:**   + using mathematical arguments to convince   + includes anticipating consequences * **decisions:**   + Have students explore which of two scenarios they would choose and then defend their choice. * **many ways:**   + including oral, written, visual, use of technology   + communicating effectively according to what is being communicated and to whom * **Represent:**   + using models, tables, graphs, words, numbers, symbols   + connecting meanings among various representations * **discussions:**   + partner talks, small-group discussions, teacher-student conferences * **discourse:**   + is valuable for deepening understanding of concepts   + can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions * **Reflect:**    + share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions * **Connect mathematical concepts:**   + to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration) * **mistakes:**   + range from calculation errors to misconceptions * **opportunities to advance learning:**   + by:     - analyzing errors to discover misunderstandings     - making adjustments in further attempts     - identifying not only mistakes but also parts of a solution that are correct * **Incorporate:**   + by:     - collaborating with Elders and knowledge keepers among local First Peoples     - exploring the [First Peoples Principles of Learning](http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf) (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)     - making explicit connections with learning mathematics     - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections * **knowledge:**   + local knowledge and cultural practices that are appropriate to share and that are non-appropriated * **practices:**   + [Bishop’s cultural practices](http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm): counting, measuring, locating, designing, playing, explaining   + [Aboriginal Education Resources](http://www.aboriginaleducation.ca/)   + [*Teaching Mathematics in a First Nations Context*,](http://www.fnesc.ca/resources/math-first-peoples/) FNESC |

| **MATHEMATICS – Calculus  Content – Elaborations Grade 12** |
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| * **functions:**   + parent functions from Pre-calculus 12   + piecewise functions   + inverse trigonometric functions * **limits:**   + from table of values, graphically, and algebraically   + one-sided versus two-sided   + end behaviour   + intermediate value theorem * **differentiation:**   + history   + definition of derivative   + notation * **rate of change:**   + average versus instantaneous   + slope of secant and tangent lines * **differentiation rules:**   + power, product; quotient and chain   + transcendental functions: logarithmic, exponential, trigonometric * **applications:**   + relating graph of *f(x)* to *f’(x)* and *f”(x)*   + increasing/decreasing, concavity   + differentiability, mean value theorem   + Newton’s method   + problems in contextual situations, including related rates and optimization problems * **integration:**   + definition of an integral   + notation   + definite and indefinite * **approximations:**   + Riemann sum, rectangle approximation method, trapezoidal method * **methods of integration:**   + antiderivatives of functions   + substitution   + by parts * **applications:**   + area under a curve, volume of solids, average value of functions   + differential equations   + initial value problems   + slope fields |