**Area of Learning: MATHEMATICS — Calculus Grade 12**

**BIG IDEAS**

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| The **concept of a limit** is foundational to calculus. |  | Differential calculus develops the concept of **instantaneous rate of change**. |  | Integral calculus develops the concept of determining a product involving a **continuously changing** quantity over an interval. |  | Derivatives and integrals are **inversely related**. |

**Learning Standards**

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| **Curricular Competencies** | **Content** |
| *Students are expected to do the following:*Reasoning and modelling* Develop **thinking strategies** to solve puzzles and play games
* Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools**
* **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number
* **Model** with mathematics in **situational contexts**
* **Think creatively** and with **curiosity and wonder** when exploring problems

Understanding and solving* Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving
* **Visualize** to explore and illustrate mathematical concepts and relationships
* Apply **flexible and strategic approaches** to **solve problems**
* Solve problems with **persistence and a positive disposition**
* Engage in problem-solvingexperiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures
 | *Students are expected to know the following:** **functions** and graphs
* **limits:**
	+ left and right limits
	+ limits to infinity
	+ continuity
* **differentiation:**
	+ **rate of change**
	+ **differentiation rules**
	+ higher order, implicit
	+ **applications**
* **integration:**
	+ **approximations**
	+ fundamental theorem of calculus
	+ **methods of integration**
	+ **applications**
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**Area of Learning: MATHEMATICS — Calculus Grade 12**

**Learning Standards (continued)**

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| **Curricular Competencies** | **Content** |
| Communicating and representing* **Explain and justify** mathematical ideas and **decisions** in **many ways**
* **Represent** mathematical ideas in concrete, pictorial, and symbolic forms
* Use mathematical vocabulary and language to contribute to **discussions** in the classroom
* Take riskswhen offering ideas in classroom **discourse**

Connecting and reflecting* **Reflect** on mathematical thinking
* **Connect mathematical concepts** with each other, other areas, and personal interests
* Use **mistakes** as **opportunities to advance learning**
* **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with computer science concepts
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|  **MATHEMATICS – Calculus Big Ideas – Elaborations Grade 12** |
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| * **concept of a limit:**
	+ Differentiation and integration are defined using limits.

*Sample questions to support inquiry with students:** + Why is a limit useful?
	+ How can we use historical examples (e.g., Achilles and the tortoise) to describe a limit?
* **instantaneous rate of change:**
	+ developing rate of change from average to instantaneous

*Sample questions to support inquiry with students:** + How can a rate of change be instantaneous?
	+ When do we use rate of change?
* **continuously changing:**
	+ area (height x width) under a curve where the height of the region is changing; volume of a solid (area x length) where cross-sectional area is changing; work (force x distance) where force is changing
	+ Finding these products requires finding an infinite sum.

*Sample questions to support inquiry with students:** + What is the value of using rectangles to approximate the area under a curve?
	+ Why is the fundamental theorem of calculus so fundamental?
* **inversely related:**
	+ The fundamental theorem of calculus describes the relationship between integrals and antiderivatives.

*Sample questions to support inquiry with students:** + How are derivatives and integrals related?
	+ Why are antiderivatives important?
	+ What is the difference between an antiderivative and an integral?
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|  **MATHEMATICS – Calculus Curricular Competencies – Elaborations Grade 12** |
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| * **thinking strategies:**
	+ using reason to determine winning strategies
	+ generalizing and extending
* **analyze:**
	+ examine the structure of and connections between mathematical ideas (e.g., limits, derivatives, integrals)
* **reason:**
	+ inductive and deductivereasoning
	+ predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
* **technology:**
	+ graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
	+ can be used for a wide variety of purposes, including:
		- exploring and demonstrating mathematical relationships
		- organizing and displaying data
		- generating and testing inductive conjectures
		- mathematical modelling
* **other tools:**
	+ manipulatives such as algebra tiles and other concrete materials
* **Estimate reasonably:**
	+ be able to defend the reasonableness of an estimate across mathematical contexts
* **fluent, flexible, and strategic thinking:**
	+ includes:
		- using known facts and benchmarks, partitioning, applying number strategies to approximate limits, derivatives, and integrals
		- choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?)
* **Model:**
	+ use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
	+ take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
* **situational contexts:**
	+ including real-life scenarios and open-ended challenges that connect mathematics with everyday life
* **Think creatively:**
	+ by being open to trying different strategies
	+ refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music
* **curiosity and wonder:**
	+ asking questions to further understanding or to open other avenues of investigation
* **inquiry:**
	+ includes structured, guided, and open inquiry
	+ noticing and wondering
	+ determining what is needed to make sense of and solve problems
* **Visualize:**
	+ create and use mental images to support understanding
	+ Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
* **flexible and strategic approaches:**
	+ deciding which mathematical tools to use to solve a problem
	+ choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
* **solve problems:**
	+ interpret a situation to identify a problem
	+ apply mathematics to solve the problem
	+ analyze and evaluate the solution in terms of the initial context
	+ repeat this cycle until a solution makes sense
* **persistence and a positive disposition:**
	+ not giving up when facing a challenge
	+ problem solving with vigour and determination
* **connected:**
	+ through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
	+ by posing and solving problems or asking questions about place, stories, and cultural practices
* **Explain and justify:**
	+ using mathematical arguments to convince
	+ includes anticipating consequences
* **decisions:**
	+ Have students explore which of two scenarios they would choose and then defend their choice.
* **many ways:**
	+ including oral, written, visual, use of technology
	+ communicating effectively according to what is being communicated and to whom
* **Represent:**
	+ using models, tables, graphs, words, numbers, symbols
	+ connecting meanings among various representations
* **discussions:**
	+ partner talks, small-group discussions, teacher-student conferences
* **discourse:**
	+ is valuable for deepening understanding of concepts
	+ can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions
* **Reflect:**
	+ share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
* **Connect mathematical concepts:**
	+ to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
* **mistakes:**
	+ range from calculation errors to misconceptions
* **opportunities to advance learning:**
	+ by:
		- analyzing errors to discover misunderstandings
		- making adjustments in further attempts
		- identifying not only mistakes but also parts of a solution that are correct
* **Incorporate:**
	+ by:
		- collaborating with Elders and knowledge keepers among local First Peoples
		- exploring the [First Peoples Principles of Learning](http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf) (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
		- making explicit connections with learning mathematics
		- exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
* **knowledge:**
	+ local knowledge and cultural practices that are appropriate to share and that are non-appropriated
* **practices:**
	+ [Bishop’s cultural practices](http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm): counting, measuring, locating, designing, playing, explaining
	+ [Aboriginal Education Resources](http://www.aboriginaleducation.ca/)
	+ [*Teaching Mathematics in a First Nations Context*,](http://www.fnesc.ca/resources/math-first-peoples/) FNESC
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|  **MATHEMATICS – Calculus Content – Elaborations Grade 12** |
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| * **functions:**
	+ parent functions from Pre-calculus 12
	+ piecewise functions
	+ inverse trigonometric functions
* **limits:**
	+ from table of values, graphically, and algebraically
	+ one-sided versus two-sided
	+ end behaviour
	+ intermediate value theorem
* **differentiation:**
	+ history
	+ definition of derivative
	+ notation
* **rate of change:**
	+ average versus instantaneous
	+ slope of secant and tangent lines
* **differentiation rules:**
	+ power, product; quotient and chain
	+ transcendental functions: logarithmic, exponential, trigonometric
* **applications:**
	+ relating graph of *f(x)* to *f’(x)* and *f”(x)*
	+ increasing/decreasing, concavity
	+ differentiability, mean value theorem
	+ Newton’s method
	+ problems in contextual situations, including related rates and optimization problems
* **integration:**
	+ definition of an integral
	+ notation
	+ definite and indefinite
* **approximations:**
	+ Riemann sum, rectangle approximation method, trapezoidal method
* **methods of integration:**
	+ antiderivatives of functions
	+ substitution
	+ by parts
* **applications:**
	+ area under a curve, volume of solids, average value of functions
	+ differential equations
	+ initial value problems
	+ slope fields
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